Three solutions for elliptic Dirichlet boundary value problem with singular weight.

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We consider a p-Laplacian Dirichlet boundary value problem with a Hardy-type singular weight on a bounded Lipschitz domain $\Omega \subseteq \mathbb{R}^n$ ($0 \in \Omega$, $2 \leq p < n$). Specifically, we seek weak solutions $u \in W_0^{1,p}(\Omega)$ of the problem

$$-\Delta_p u + \mu \frac{|u|^{p-2}u}{|x|^p} = \lambda f(u) + \gamma g(u) \quad \text{in } \Omega, \qquad u\big|_{\partial\Omega} = 0,$$

where $\mu > 0$ and $\lambda, \gamma \geq 0$ are parameters. We assume that $f\colon \mathbb{R} \to \mathbb{R}$ is continuous, satisfies sublinearity conditions at zero and at infinity, and that its primitive $F(t) = \int_0^t f(s)\,ds$ attains positive values. The function $g\colon \mathbb{R} \to \mathbb{R}$ is assumed to have subcritical growth. Under these assumptions, applying the Ricceri three critical points theorem together with the Hardy inequality and the Browder–Minty theorem for uniformly monotone operators, we prove the existence of at least three weak solutions (two of them non-trivial) for λ belonging to some compact interval $[a, b] \subset (\beta, +\infty)$ and sufficiently small $\gamma \geq 0$.

References

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