Well-posedness analysis of a unilateral contact problem

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We consider a mathematical model that describes the equilibrium of a nonlinear elastic membrane which can arrive in contact with a rigid obstacle, the so-called foundation. The membrane is fixed on its boundary, is acted upon by a vertical force, the contact is frictionless and is modelled with the well-known Signorini boundary condition. The novelty is that we model the material's behaviour with a Hencky-type elastic constitutive law and, therefore, the problem is governed by a p-Laplacian operator. Using such a constitutive assumption makes the model nonstandard from a mechanical point of view and challenging from a mathematical point of view. We use a Weierstrass-type minimization argument to prove the unique weak solvability of the model. Then, we turn to the well-posedness analysis of the problem, which represents the main novelty of this manuscript. Thus, we introduce three well-posedness concepts, compare them and state and prove strong and weak well-posedness results. We also discuss the problem of finding an optimal well-posedness concept. Finally, we apply these results to prove that the solution depends continuously with respect to the density of applied forces and the initial gap.

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