Buckling of a Gao Beam under Combined Axial and Vertical Loading

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We investigate the buckling and post-buckling behaviour of the nonlinear Gao beam subjected to combined axial and vertical loading. The model introduced by Gao [1] extends the classical Euler–Bernoulli beam theory by incorporating a nonlinear term that makes it possible to describe moderately large deflections and nonconvex post-buckling regimes. The problem is formulated variationally through the corresponding potential energy functional, whose convexity properties determine the number and character of equilibrium solutions.

The main attention is paid to the influence of a prescribed vertical load on the critical axial load. For zero vertical loading, the critical value coincides with the classical Euler buckling load, in agreement with the purely axial case studied in [3]. For nonzero vertical loading, however, the critical axial load increases and the symmetric pitchfork bifurcation changes into an imperfect pitchfork bifurcation [2]. We describe critical curves separating parameter regions with a unique equilibrium solution from regions with three equilibrium solutions. These branches correspond to a global minimum, a local minimum and a saddle point of the potential energy functional.

The theoretical observations are complemented by numerical experiments for cantilever and simply supported beams under uniform and point vertical loads. We also discuss how material and geometric parameters, including Young's modulus, Poisson's ratio, beam thickness, width and length, affect critical loads, maximum deflections and energy values. The results contribute to a clearer understanding of the post-buckling response of Gao beams under realistic combined loading conditions.

Keywords: Gao beam; buckling; post-buckling analysis; combined loading; nonlinear beam model; imperfect bifurcation; variational formulation.

References

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