Multiplicity Results for the Nonlinear Weighted Laplacian Equation on the Sierpiński Gasket

Krzysztof Jelito, Michał Bełdziński

We establish results concerning the existence of solutions to the nonlinear equation driven by the weighted Laplacian with the Dirichlet boundary condition on the Sierpinski gasket V:

$$\begin{cases} -\Delta_a u(\boldsymbol{x}) \ni f\big(\boldsymbol{x}, u(\boldsymbol{x})\big) & \text{for } \boldsymbol{x} \in V \setminus V_0, \\ u(\boldsymbol{x}) = 0 & \text{for } \boldsymbol{x} \in V_0, \end{cases} \tag{1}$$

where V_0 is the intrinsic boundary of V, and the function $f : V \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function satisfying suitable growth assumptions that ensure well-posedness of the problem. The function $a : V \to (0, \infty)$ is the bounded weight function defining the weighted Laplacian $-\Delta^a$. The existence of weak solutions is obtained using variational methods, including the Direct Method of the Calculus of Variations and the Mountain Pass Lemma. Additional growth conditions on the nonlinearity term are imposed to establish the existence of nontrivial and multiple solutions. The multiplicity results are inspired by the approach of Bereanu, Jebelean, and Mawhin in [2] and are adapted to the fractal setting of the Sierpiński gasket.

A weak solution of (1) is defined as a function satisfying the hemivariational inequality

$$\int_V f(\boldsymbol{x}, u(\boldsymbol{x}))\big(v(\boldsymbol{x}) - u(\boldsymbol{x})\big)\, d\mu(\boldsymbol{x}) + \tfrac{1}{2}\mathcal{E}_a(v) \geq \tfrac{1}{2}\mathcal{E}_a(u) \quad \text{for all } v \in H_0^1(V),$$

where $\mathcal{E}_a$ denotes the weighted energy functional associated with the weighted Laplacian $-\Delta_a$.

References

[1] Bełdziński, M. and Jelito, K., *Multiplicity Results for the Nonlinear Weighted Laplacian Equation on the Sierpiński Gasket*, arXiv preprint arXiv:2606.12678, 2026.DOI: 10.48550/arXiv.2606.12678.

[2] Bereanu, C., Jebelean, P., and Mawhin, J. *Multiple solutions for Neumann and periodic problems with singular φ-Laplacian.* Nonlinear Analysis, 75:731–740, 2012.

First Author: Michał, Bełdziński
Affiliation: *Institute of Mathematics, Lodz University of Technology*
93-590, Poland
e-mail: michal.beldzinski@p.lodz.pl

Second Author: Krzysztof, Jelito
Affiliation: *Institute of Mathematics, Lodz University of Technology*
93-590, Poland
Department of Mathematics, Silesian University of Technology
44-100, Poland
e-mail: krzysztof.jelito@dokt.p.lodz.pl