Guaranteed Bounds for the Critical Buckling Load of the Euler–Bernoulli and Gao Beam Models

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This contribution presents a computational framework for obtaining guaranteed bounds for the critical buckling load at which instability occurs in slender structures. The buckling analysis is formulated as an eigenvalue problem, where the smallest eigenvalue corresponds to the critical load. Reliable computation of this eigenvalue is thus essential for the safe design and assessment of engineering structures. While finite element approximations provide upper bounds for the eigenvalues through the min–max principle, obtaining lower bounds is more challenging. To derive guaranteed lower bounds, the proposed approach [1] combines finite element approximations with interpolation error estimates and a Wirtinger-type inequality. The method is first developed for the classical Euler-Bernoulli beam with piecewise constant bending stiffness and is subsequently extended to beams with variable bending stiffness. Finally, the approach is applied to the nonlinear Gao beam model by exploiting the equivalence of its first eigenvalue with that of the linear Euler-Bernoulli problem [2]. Several numerical examples are presented to demonstrate the accuracy and convergence rates of the proposed method.

References

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[2] H. Netuka, J. Machalová, *Post-buckling solutions for the Gao beam*, Quart.J. Mech. Appl. Math. 76(3), 329–347 (2023)

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