MaMAL: Matrix-free Multigrid Augmented Lagrangian Method for Contact

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High-performance computing is essential for efficiently solving large-scale contact problems. Simulating such phenomena at engineering scale is often limited by computational resources, making it crucial to design algorithms that fully exploit modern hardware such as multi-core CPUs and GPUs. Iterative solvers and preconditioners play a central role in this efficiency.

Monotone multigrid methods offer optimal complexity and robustness for variational inequalities arising in contact mechanics. In parallel, augmented Lagrangian formulations provide a flexible way to handle over-constrained and fuzzy constraints while retaining a clear variational structure.

We introduce MaMAL, a Matrix-free Multigrid Augmented Lagrangian method for large-scale contact problems. The method combines multilevel nonlinear smoothing, constraint-aware coarse operators, and augmented Lagrangian updates in a matrix-free framework designed for modern GPU architectures.

Our implementation uses matrix-free differential operators and memory-efficient semi-structured meshes to discretize elasticity equations. We present the MaMAL algorithm with emphasis on nonlinear smoothing, multiplier updates, constraint coarsening, and the interaction between augmentation parameters and multigrid convergence.

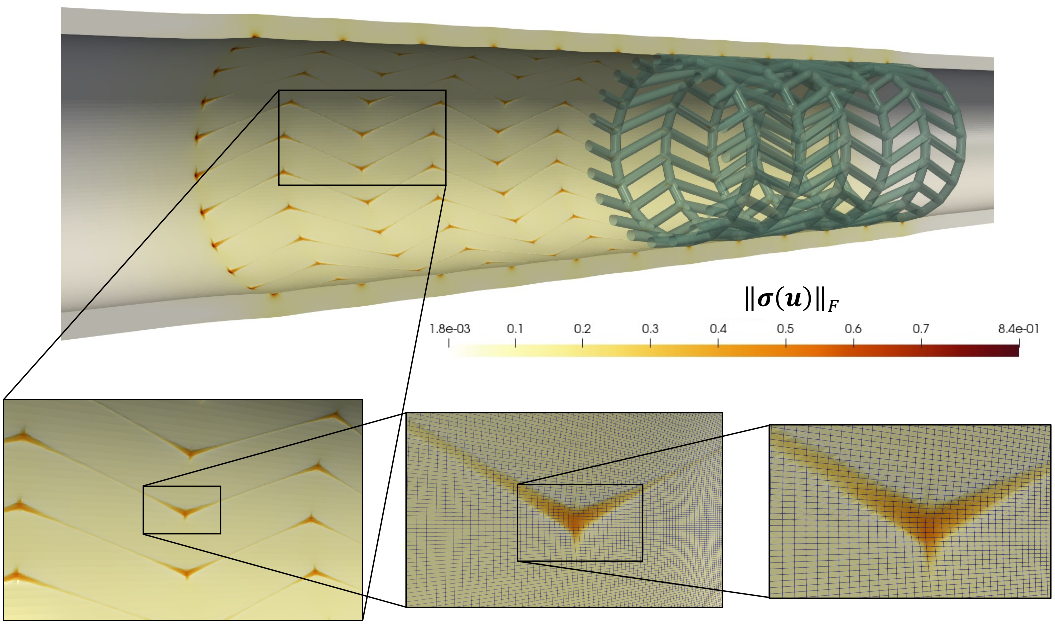

Figure 1: Stress distribution and local mesh resolution for a nonlinear contact computation.

We evaluate performance on the Grace-Hopper superchip of the CSCS Alps supercomputer. Emphasis is placed on single-node GPU performance, kernel design, and convergence behavior in representative contact scenarios. Finally, we demonstrate MaMAL's scalability on large-scale problems with hundreds of millions of degrees of freedom.

References

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