Optimal threshold values and local minimizers

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In this talk, we investigate the role of threshold parameters in the existence of local minimizers for functionals of the form $\Phi - \lambda\Psi$. We present new characterizations showing that the qualitative properties of local minimizers vary with the parameter λ. These results provide a unified framework that applies to a variety of problems, including second-order Dirichlet boundary value problems. More precisely, consider the Dirichlet problem

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Under suitable assumptions on f, one can define a threshold value λ^* in terms of the associated energy functional. Then, if

$$\lambda \in (0, \lambda^*),$$

the above problem admits solutions corresponding to local minimizers, while for

$$\lambda > \lambda^*,$$

no solution exists. In particular, we show that there exists $\hat{\lambda} \leq \lambda^*$ such that, for every $\lambda \in (0, \hat{\lambda})$, the corresponding solutions are local minimum points which are global on balls centered at the origin, whereas, if $\hat{\lambda} < \lambda^*$, for every $\lambda \in (\hat{\lambda}, \lambda^*)$, they are local minimizers which are not global on any ball centered at the origin.

References

[1] L. Dupaigne, *Stable Solutions of Elliptic Partial Differential Equations.* Chapman & Hall/CRC Monographs and Surveys in Pure and Applied Mathematics, 143 (2011)

[2] G. Bonanno, *A critical point theorem via the Ekeland variational principle*, Nonlinear Anal. 75, 2992–3007 (2012)

[3] G. Bonanno, *Relations between the mountain pass theorem and local minima*, Adv. Nonlinear Anal. 1, 205–220 (2012)

[4] G. Bonanno and P. Candito, *Local minimizers and optimal thresholds for functionals arising from nonlinear differential problems*, submitted.

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