

Emerging Trends in Applied Mathematics and Mechanics

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ETAMM

Emerging Trends in Applied
Mathematics and Mechanics

Plenary lectures featuring:
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Gabriele Bonanno,
Noël Challamel,
Wojciech Kryszewski, Vicentiu
Radulescu, Stanisław Stupkiewicz

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PLENARY SESSION

Genesis, Degeneracy and Progress in Nonlocal Structural Mechanics

Raffaele Barretta

Nonlocal continuum mechanics still faces open questions concerning applicability of integral constitutive theories to structural modelling. This lecture aims to provide an updated overview of nonlocal continuum problems, addressing challenges and progress. Major advancements and recent contributions in nonlocal mechanics are presented and critically discussed.

The well-known strain-driven formulation introduced by Eringen [1] revealed a fundamental incompatibility between the constitutive relations and equilibrium conditions when applied to structural problems. This ill-posedness, widely recognized for bounded continua [2], has been recently proven also in the context of unbounded domains [3] on the basis of regularity properties of spatial convolutions, as investigated in [4]. A consistent approach, amending Eringen model, was proposed in [2] based on stress-driven spatial convolution. Further issues concern interactions with nonlocal elastic foundations, for which consistent improvements have recently been proposed [7].

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Optimal threshold values and local minimizers

Gabriele Bonanno

In this talk, we investigate the role of threshold parameters in the existence of local minimizers for functionals of the form $\Phi - \lambda\Psi$. We present new characterizations showing that the qualitative properties of local minimizers vary with the parameter λ . These results provide a unified framework that applies to a variety of problems, including second-order Dirichlet boundary value problems. More precisely, consider the Dirichlet problem

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Under suitable assumptions on f , one can define a threshold value λ^* in terms of the associated energy functional. Then, if

$$\lambda \in (0, \lambda^*),$$

the above problem admits solutions corresponding to local minimizers, while for

$$\lambda > \lambda^*,$$

no solution exists. In particular, we show that there exists $\hat{\lambda} \leq \lambda^*$ such that, for every $\lambda \in (0, \hat{\lambda})$, the corresponding solutions are local minimum points which are global on balls centered at the origin, whereas, if $\hat{\lambda} < \lambda^*$, for every $\lambda \in (\hat{\lambda}, \lambda^*)$, they are local minimizers which are not global on any ball centered at the origin.

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Analytical and numerical approaches in nonlocal and peridynamic structural mechanics

Noël Challamel

This plenary lecture is devoted to the development of size-dependent structural mechanics through integral models, also classified as strongly nonlocal approaches. The presentation focuses on the two main branches of strongly nonlocal elasticity: the strain-driven nonlocal approach and the much more recent peridynamic approach, also called the relative displacement integral approach. Using one-dimensional problems, applied to bar or beam theories in both statics and dynamics, it is shown that the two integral approaches can be ill-posed for certain exponential kernels. Both problems can be regularized by combining a local and a nonlocal measure, leading to a well-posed integro-differential equation.

The idea of mixing local and nonlocal elasticity has been extensively developed for two-phase strain-driven nonlocal models. Combining local and peridynamic elasticity was developed later by Di Paola and collaborators. Exact solutions for two-phase curvature-driven nonlocal beams with normalized exponential kernel have been derived in earlier works, while exact solutions for two-phase peridynamic beams with exponential kernel are more recent. It has also been shown that the two-phase peridynamic beam model can be reformulated as a two-phase curvature-driven beam model for specific kernels.

Other nonlocal approaches applied to beam problems are presented, such as the nonlocal strain gradient beam model, the micromorphic beam model, the nonlocal stress-driven beam model, the two-phase nonlocal stress-driven beam model, and the nonlocal stress gradient beam model. For some exponential kernels, it is possible to merge the nonlocal strain gradient and nonlocal stress gradient beam models, thereby allowing reconciliation of stress- and strain-based formulations. Both equivalent formulations can capture the so-called softening/stiffening small length scale effects.

The presentation concludes with a numerical study of the associated nonlocal eigenvalue differential problem, using several methods such as the finite difference method, the finite element method, the iterative variational method, and the exact finite element method. The study of nonlocal and peridynamic eigenvalue problems in 2D and 3D is also addressed.

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From Perov Contractions to Nonexpansive Mappings: A Spectral Approach to Fixed Points in Product Spaces

Wojciech Kryszewski

We study fixed point problems for mappings on finite products of Banach spaces satisfying Lipschitz estimates controlled by a nonnegative matrix A . Our aim is to clarify the relation between Perov-type matrix contractions and classical scalar fixed point principles. If $r(A) < 1$, we show that a suitable lattice norm on the coordinate space makes the corresponding subordinate matrix norm strictly smaller than one; hence the vector-valued contraction becomes an ordinary contraction in an equivalent scalar metric, and Perov's theorem follows from the Banach contraction principle. For irreducible matrices, weighted ℓ_∞ - and ℓ_1 -type norms determined by Perron vectors realize the spectral radius exactly. We then study the critical case $r(A) = 1$. In the irreducible case, an adapted Perron-Frobenius norm makes the mapping nonexpansive, allowing fixed point existence to be obtained by normal-structure methods. In the reducible case, Frobenius normal form yields a block triangular decomposition and reduces the problem successively to fixed point problems associated with the diagonal blocks. This connects Perov-type contraction theory with classical contraction and nonexpansive fixed point theory through the spectral properties of the Lipschitz matrix.

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Non-autonomous (p, q) -equations and some perspectives

Vicențiu D. Rădulescu

This talk is concerned with the qualitative analysis of several classes of unbalanced non-autonomous equations. In the first part of my talk, I shall develop an exhaustive analysis for the nonautonomous (p, q) -eigenvalue problem with indefinite weight and lack of compactness. This analysis distinguishes between the cases where $p < q$ or $q < p$. In the first case, we establish the existence of a continuous family of eigenvalues, starting from the principal frequency of a suitable single phase eigenvalue problem. In the latter case, we prove the existence of a discrete family of positive eigenvalues, which diverges to infinity.

In the second part of my talk, I will discuss a nonlinear Dirichlet problem driven by a nonautonomous double-phase differential operator and with a reaction consisting of a strongly singular term plus a concave perturbation. The proofs combine the Nehari manifold method with energy estimates and related variational and topological arguments.

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Modelling of Transformation Patterns in Pseudoelastic Shape Memory Alloys

Stanisław Stupkiewicz

Propagating instabilities and the corresponding transformation patterns are commonly observed in polycrystalline shape memory alloys, in particular in pseudoelastic NiTi alloys under predominantly tensile loading. These effects result from a non-monotonic intrinsic material response, which leads to strain localization and propagation of macroscopic phase transformation fronts. The deformation patterns then resemble Lüders bands in plasticity, although more complex patterns are also observed, and the overall load-displacement response features a characteristic plateau. In order to model related effects, we developed a gradient-enhanced thermomechanical model of pseudoelasticity. The model introduces a characteristic length scale and provides regularization. A micromorphic formulation is employed to facilitate finite-element implementation. The finite-strain model includes a number of important features, notably tension-compression asymmetry, transverse isotropy of the transformation strain, and strain-dependent hardening-softening response. The model is formulated within the incremental energy minimization framework and is able to describe several experimentally observed effects, including the effect of loading rate on martensite bands and the formation of multi-prong interfaces in NiTi tubes subjected to combined tension and torsion.

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MS1

Variational and monotonicity methods in PDEs with applications

Mathematical Analysis of a Contact Problem for a Non-Clamped Nonlinear Viscoelastic Rod

Krzysztof Bartosz

We investigate the dynamic behavior of a nonlinear viscoelastic rod in contact with a deformable foundation along its length and with a reactive obstacle at one end. The rod is subjected to body forces that govern its mechanical response over time. We establish the existence of a weak solution to the resulting contact problem. The proof relies on a time discretization scheme for second-order differential inclusions involving multivalued pseudomonotone operators. The model extends an earlier formulation by removing the homogeneous Dirichlet boundary condition at the left end of the rod. This modification allows for a more general mechanical setting in which the rod is not clamped. At the same time, the principal operator is no longer coercive, which substantially increases the analytical difficulty of the problem. We overcome this issue and prove the existence of a weak solution under reasonable assumptions.

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Convergence and Well-Posedness of Nonlinear Elliptic Systems

M.-M. Boureanu, L. Dafinoiu, M. Sofonea

We are concerned with the analysis of a nonlinear system formulated in real Hilbert spaces, governed by two sets of constraints and four bilinear forms. We first prove the unique solvability of the system using standard techniques from the theory of elliptic variational inequalities. We then provide a convergence criterion for its solution in terms of necessary and sufficient conditions. Furthermore, we introduce two notions of well-posedness and compare them. Finally, we present a one-dimensional example illustrating the theoretical results and highlighting the difference between these two notions of well-posedness.

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Infinitely Many Solutions for Higher-Order Problems

M.-M. Boureanu, A. Chinnì, and B. Di Bella

We are concerned with the weak solvability of fourth-order elliptic problems involving variable exponents and Navier boundary conditions, driven by Leray-Lions type operators. We work on a class of extension domains, which encompasses not only smooth domains but also nonsmooth structures, such as fractals. By relying on critical point theory, we deduce the existence of infinitely many weak solutions to the problem under investigation. Moreover, we illustrate the generality of our study through various examples.

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Weak Solutions of Elliptic Problems in Exterior Domains

Wiktor Burakowski

Our aim is to obtain the existence of a weak solution for a class of elliptic problems in exterior domains. We assume that one coefficient is measurable and is approximated by a pointwise convergent sequence, while another nonlinearity is locally Hölder continuous. We also allow the case in which the product nonlinearity can be negative at the origin. A starting point is, for every approximating problem, the existence of a classical positive bounded solution. Then we consider a certain auxiliary sequence of solutions and show that it has a subsequence converging weakly to a function which is a weak solution of the original problem.

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Homogenisation of fractional diffusion problems

Andreas Buchinger, Krešimir Burazin, Ivana Crnjac, Marko Erceg, Maja Jolić and
Marcus Waurick

Fractional differential operators have become an important tool for modelling nonlocal phenomena, including anomalous diffusion, image processing, and continuum mechanics models such as peridynamics. We study homogenisation of fractional divergence form diffusion problems with highly oscillatory coefficients. We characterise the convergence of these coefficients in terms of two classical notions of convergence: H -convergence, describing the local behaviour of the operator, and weak- $*$ convergence, accounting for the nonlocal effects induced by the fractional structure. We further establish several fundamental properties of this convergence, including uniqueness of limits, metrisability, energy convergence. Moreover, we analyse the limiting regimes of finite-horizon fractional diffusion problems.

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Multiple Solutions for the p -Laplacian Equation with Convection

Giuseppe Failla, Marek Galewski and Leszek Gasiński

We investigate in [1] the following class of nonlinear elliptic problems involving a p -Laplacian operator and a gradient-dependent reaction term, which introduces a convective component and disrupts the variational structure of the problem, i.e.,

$$\begin{cases} -\Delta_p u = f(x, u, \nabla u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (\text{P})$$

being $\Omega \subset \mathbb{R}^N$, $N \geq 2$, a bounded domain with smooth boundary and $1 < p < N$. On the reaction term, we assume that

(H) $f : \Omega \times \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{R}$ is a Carathéodory function, such that

- (i) $f(x, 0, \xi) = 0$ for a.a. $x \in \Omega$, for all $\xi \in \mathbb{R}^N$;
- (ii) there exists $\ell \in [1, p]$, such that

$$|f(x, t, \xi)| \leq c(1 + |t|^{\ell-1})$$

for a.a. $x \in \Omega$, for all $t \in \mathbb{R}$, for all $\xi \in \mathbb{R}^N$, being c a positive constant. In addition, if $\ell = p$ we require that $c < \lambda_1$, being λ_1 the first eigenvalue of $(-\Delta_p, W_0^{1,p}(\Omega))$;

- (iii) there exist c_0, c_1 positive constants and $s \leq p \leq r < p^*$, $s \neq r$, such that

$$f(x, t, \xi)t \geq c_0|t|^s - c_1|t|^r$$

for a.a. $x \in \Omega$ and for all $t \in \mathbb{R}$, $\xi \in \mathbb{R}^N$;

- (iv) there exist $\zeta_{\pm} \in W^{1,p}(\Omega) \cap C(\overline{\Omega})$ and constants c_-, c_+ such that

$$\begin{aligned} \zeta_-(x) &\leq c_- < 0 < c_+ \leq \zeta_+(x) \text{ for a.a. } x \in \Omega; \\ f(x, \zeta_+(x), \xi) &\leq f(x, \zeta_-(x), \xi) \text{ for a.a. } x \in \Omega, \text{ for all } \xi \in \mathbb{R}^N. \end{aligned}$$

- (v) there exist $\mu \in (1, \ell)$ and $\delta_0 > 0$ such that

$$0 < f(z, s, \xi)s \leq \mu F(x, s, \xi)$$

for a.a. $x \in \Omega$, for all $0 < |s| \leq \delta_0$, $x \in \mathbb{R}^N$ and $\text{essinf}_{\Omega \times \mathbb{R}^N} F(\cdot, \delta_0, \cdot) > 0$.

Motivated by recent advances in the field, see e.g. [2, 3], we establish the existence of at least two weak solutions with sign information—one positive and one negative—without imposing asymptotic conditions near zero. Our approach relies on variational techniques on a counterpart of problem (P) with frozen gradient term, and the Leray–Schauder alternative principle, extending previous results and pertaining to a broader class of nonlinearities.

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Equilibria on Simplices in Hilbert Spaces

Karolina Fisiak

I will present and discuss equilibrium existence results for continuous maps defined on compact and convex subsets of Hilbert spaces and bring up theorems in which the existence of zeros stems from assumptions regarding boundary behaviour of mappings. The main subject will revolve around Browder theorem, Vrahatis theorem and its infinite-dimensional version. During the presentation I will examine tangent and normal cones and their connection to the orthogonal projection. Additionally, I will show that sometimes the sign conditions can be interpreted from the perspective of the geometric property of belonging to tangent and normal cones.

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Existence and Location of the Solutions to Singular Elliptic Problems in the Exterior Domain

Krzysztof Garbowski

We present results regarding the existence of positive solutions for singular elliptic problems in exterior domains with boundary condition at a sphere and decay at infinity. In the first part of the presentation, we discuss the history of similar problems and the existence of classical solutions in a nonsingular case. Our main tool is the Noussair-Swanson theorem concerning the sub-supersolution approach. Having explicit formulas for both sub- and supersolutions, we provide the location of the solution depending on the boundary datum. In the second part, we show the existence of a positive solution to the singular problem using the sub-supersolution method for bounded subdomains, the unbounded-domain approximation method, and a classical convergence procedure. A location theorem is also presented for the singular case.

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Strong Solutions to Singular Discontinuous p-Laplacian Problems

Umberto Guarnotta and Salvatore A. Marano

The talk is devoted to the existence of solutions to a p-Laplacian problem whose reaction is singular, that is, it blows up when the solution approaches zero, and possesses a null-measure set of discontinuity points. The techniques presented are based on regularization arguments, variational methods, regularity theory, and locality properties. Moreover, a problem exhibiting discontinuous convection terms, namely terms depending on the gradient of the solution, will be presented. In this case, existence of solutions is obtained via monotonicity techniques.

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Multiplicity Results for the Nonlinear Weighted Laplacian Equation on the Sierpiński Gasket

Krzysztof Jelito, Michał Beldziński

We establish results concerning the existence of solutions to the nonlinear equation driven by the weighted Laplacian with the Dirichlet boundary condition on the Sierpinski gasket V :

$$\begin{cases} -\Delta_a u(\mathbf{x}) \ni f(\mathbf{x}, u(\mathbf{x})) & \text{for } \mathbf{x} \in V \setminus V_0, \\ u(\mathbf{x}) = 0 & \text{for } \mathbf{x} \in V_0, \end{cases} \quad (1)$$

where V_0 is the intrinsic boundary of V , and the function $f : V \times \mathbb{R} \rightarrow \mathbb{R}$ is a Carathéodory function satisfying suitable growth assumptions that ensure well-posedness of the problem. The function $a : V \rightarrow (0, \infty)$ is the bounded weight function defining the weighted Laplacian $-\Delta^a$. The existence of weak solutions is obtained using variational methods, including the Direct Method of the Calculus of Variations and the Mountain Pass Lemma. Additional growth conditions on the nonlinearity term are imposed to establish the existence of nontrivial and multiple solutions. The multiplicity results are inspired by the approach of Bereanu, Jebelean, and Mawhin in [2] and are adapted to the fractal setting of the Sierpiński gasket.

A weak solution of (1) is defined as a function satisfying the hemivariational inequality

$$\int_V f(\mathbf{x}, u(\mathbf{x}))(v(\mathbf{x}) - u(\mathbf{x})) d\mu(\mathbf{x}) + \frac{1}{2}\mathcal{E}_a(v) \geq \frac{1}{2}\mathcal{E}_a(u) \quad \text{for all } v \in H_0^1(V),$$

where $\mathcal{E}_a$ denotes the weighted energy functional associated with the weighted Laplacian $-\Delta_a$.

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Existence and Regularity for Certain Quasilinear Problem Involving Elliptic Operator in the Nondivergent Form

Agnieszka Kałamajska

Let $\gamma > 0$, $\Omega \subseteq \mathbb{R}^n$ be an open domain with sufficiently regular boundary, $f \in L^1(\Omega)$, and P be an elliptic operator in the nondivergent form. We are interested in solvability and regularity for variants of the Emden-Fowler type equation $Pu = f u^\gamma$ in Ω , $u > 0$ in Ω , $u \equiv 0$ on $\partial\Omega$. One of the tools will be a multiplicative inequality involving a certain transform of f , denoted $T_h(f)$. We will show the application of this inequality to the existence and regularity theory for nonlinear elliptic PDEs of this type and present other results related to solvability and regularity in the study of the problem. The talk will be based on two recent joint works with Dalmil Peša and Tomáš Roskovec, and with Dalmil Peša and Artur Rutkowski.

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Equations with a Generalized Fractional Laplacian: Different Approaches

Igor Kossowski

Let Ω be an open bounded set with Lipschitz boundary. We consider a weak Dirichlet Laplacian and define a generalized operator $g(-\Delta)$ through the spectral representation of $-\Delta$ and a function g given on the spectrum. We investigate the existence of solutions to the equation $g(-\Delta)u = f(x, u)$, where f is a real-valued nonlinearity, using different approaches.

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A Critical Neumann Problem with Anisotropic p -Laplacian

Gustavo Ferron Madeira

We discuss the existence of a positive solution of a critical Neumann problem involving the anisotropic p -Laplacian. The solution obtained is a bounded $C^{1,\alpha}(\Omega)$ function which is strictly positive within the domain. This is a joint work with Olimpio Miyagaki (UFSCar) and Alannio Nóbrega (UFMG), and has been partially supported by the São Paulo Research Foundation (FAPESP), Grant No. 2022/16407-1.

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On a Mixed p -Laplacian and Fractional Laplacian Problem

Marek Majewski

We investigate the existence, uniqueness and continuous dependence on parameters for a class of mixed local–nonlocal Dirichlet boundary value problems driven by the p -Laplacian and the fractional Laplacian. The nonlinear term is allowed to depend on both the unknown function and its gradient, which makes the problem non-variational and prevents the direct application of critical point methods. Our approach is based on the theory of monotone and pseudomonotone operators together with the Browder–Minty theorem and surjectivity results for coercive pseudomonotone mappings. The paper extends the corresponding theory for mixed local–nonlocal problems involving the classical Laplacian to the quasilinear setting generated by the p -Laplacian.

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A Navier-Stokes-Darcy Hemivariational Inequality

Stanisław Migórski

The paper deals with the existence of weak solution to the steady-state Navier-Stokes-Darcy model for an incompressible fluid in a bounded domain. The no-leak boundary condition and a multivalued nonmonotone slip condition of “friction type” is assumed. The latter is governed by the generalized subgradient of a locally Lipschitz superpotential. The transmission conditions across the interface boundary involve the balance of normal forces described by the Lions condition, and the Navier slip condition formulated as the Beavers-Joseph-Saffman law. We formulate the boundary-value problem as a hemivariational inequality of elliptic type for the Navier-Stokes velocity and the Darcy pressure. We employ a method based on a surjectivity results for a pseudomonotone coercive operator in a reflexive Banach space. The existence of a weak solution under suitable assumptions involving a smallness condition is proved. The recovery of the classical model from the hemivariational inequality is demonstrated.

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Dependence of positive solutions on functional parameters for a certain class of elliptic problems

Aleksandra Orpel

We discuss the following class of elliptic semilinear equations $\Delta u(x) + F(x, u(x), v(x)) + g(x)x \cdot \nabla u(x) = 0$, for $x \in \Omega_R = \{x \in \mathbb{R}^n, \|x\| > R\}$, $n > 2$ with the condition $u(x) = \Theta(\|x\|^{2-n})$ as $\|x\| \rightarrow +\infty$. The goal of the talk is to present results concerning the existence of solutions of our problem and their dependence on functional parameters v from a subset V of a certain Hölder space. The latter result will be obtained in the case of the lack of uniqueness of a solution. We will formulate conditions guaranteeing that for each parameter from V , our problem possesses a nondecreasing sequence of solutions with finite energy.

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Localization Results for Elliptic Systems with Intrinsic Operators

Roberto Livrea, Dumitru Motreanu, Angela Sciammetta

The aim of this talk is to study a class of elliptic problems and systems characterized by the presence of intrinsic operators acting on the solutions. The main goal is to establish existence and localization results for solutions in settings where the nonlinearities depend on the solution, its gradient, and intrinsic operators acting on the solution.

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Generalized Kolmogorov System

Robert Stańczy, Dorota Bors

The existence of Lyapunov function is proved for some general dynamical system in Kolmogorov framework

$$x' = g(x)G(x, y), \quad y' = h(y)H(x, y)$$

with nonzero stationary solution under some monotonicity assumptions imposed on nonlinearities. The applications include different variations of astrophysical and predator-prey models. Moreover, the existence of heteroclinic orbit is established together with estimates for its range. The applications include both stationary radially symmetric solutions for Smoluchowski-Poisson and Tolman-Oppenheimer-Volkoff equation in Milne integrated density variables. Finally, numerics will be provided for some models for Sagittarius A* discussed recently by Crespi et al.

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Symmetry, Scaling, and Optimal Time–Frequency Concentration: Minimising the Heisenberg Uncertainty in Piecewise-Polynomial and Wavelet Dictionaries

Lorenzo de Leonardis, Alessandro Mazzocchi, and Pierluigi Vellucci

We study optimal joint localisation in time and frequency through a variational formulation of the Heisenberg uncertainty product. Starting from a hierarchy of piecewise-polynomial function classes on a fixed compact interval, we define suitable mean, variance and uncertainty operators and show that the resulting functional is invariant under translations and changes of scale. This makes it possible to compare time-frequency atoms independently of their location and dilation. We then prove a symmetry reduction result: the infimum of the uncertainty over the non-negative endpoint-vanishing class is already reached inside its even subclass. Within two concrete wavelet dictionaries generated by compactly supported polynomial profiles, the tent function is shown to be the unique minimiser, with uncertainty value

$$U = \frac{3}{10}.$$

Finally, we analyse the family of p -fold self-convolutions of the rectangular window, $\text{rect}^{\{p\}}$, and prove that their uncertainty decreases monotonically to the sharp Heisenberg bound $1/4$ as $p \rightarrow \infty$. The results provide a constructive and invariance-aware explanation of why tent functions and high-order spline-like atoms arise naturally as well-localised windows, and suggest design principles for wavelet and Gabor-type dictionaries with provably controlled time-frequency concentration.

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Homogenization theory of the Reissner-Mindlin plate equation

Krešimir Burazin, Ivan Vuković

We study the homogenization of the Reissner–Mindlin plate model

$$\begin{aligned} -\operatorname{div}(P\nabla\vec{\varphi}) + S(\vec{\varphi} + \nabla w) &= \vec{f}, \\ -\operatorname{div}(S(\vec{\varphi} + \nabla w)) &= g, \end{aligned}$$

where w is the transverse displacement and $\vec{\varphi} = (\varphi_1, \varphi_2)$ denotes the rotation of the transverse fibers. The material properties are described by the fourth-order tensor P and the matrix S , while $\vec{f}$ and g are prescribed loadings.

The mathematical foundations of homogenization in the operator framework originate from Spagnolo's concept of G-convergence and were later extended by Murat and Tartar through the theory of H-convergence. Although this theory is well established for second-order elliptic equations, considerably less attention has been devoted to plate models. In recent years, H-convergence has been successfully developed for the Kirchhoff–Love plate, which is appropriate for thin structures since it neglects transverse shear effects.

Our goal is to develop an analogous homogenization theory for the Reissner–Mindlin model, which accounts for transverse shear deformation and is therefore applicable to moderately thick plates. We propose a suitable notion of H-convergence for this system and prove its principal properties, including compactness, locality, independence of boundary conditions, and the existence of correctors.

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Constant Sign and Nodal Solutions for Double Phase Problems with Power-Type Nonlocal Nonlinearities

Bartosz Wolek

This talk is devoted to the investigation of a class of nonlocal elliptic problems involving a double phase operator. The reaction term contains a nonlinear nonlocal factor, which introduces additional analytical difficulties. In the first part of the talk, we establish the existence of two nontrivial constant-sign solutions. The proof is based on a standard variational approach involving the mountain pass theorem and the Cerami compactness condition. In the second part, we introduce an auxiliary problem in which the nonlocal term is decomposed according to the positive and negative parts of the solution. Although this auxiliary problem is not, in general, equivalent to the original one, it provides a suitable framework for applying a Nehari-type approach. This method allows us to obtain a nodal weak solution for the auxiliary problem.

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MS2

Recent developments in nonlinear analysis and its applications

Network transport with nonlinear dynamics

Jacek Banasiak and Adam Błoch

The talk's starting point is a linear metapopulation model, where the individuals of subpopulations can migrate between the nodes. Acknowledging that individuals appear at the target some time after they depart from home, we arrive at a system of ordinary differential equations with delays. We rephrase the model as a transport model on a digraph, where the delays result from the flow of individuals along the edges between the connected nodes. We show that such a model requires introducing specific dynamic conditions at the nodes relating the incoming and outgoing flows. The developed framework is an extension of the well-known dynamic network transport model and can be used to build a range of models describing many metapopulation phenomena, e.g., disease spread through migrations or cell maturation. We present a qualitative analysis of the general model using the theory of semigroups of strongly continuous operators and extend it cover nonlinear dynamics at the nodes and along the edges.

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Weak Solutions to Semilinear Elliptic Equations with Nonlocal Boundary Conditions of Dirichlet Type

Michał Beldziński, Marek Galewski, Igor Kossowski

We discuss existence results for semilinear elliptic equations subject to nonlocal Dirichlet-type boundary conditions. We first consider an auxiliary problem with a weak formulation and a nonlocal boundary term involving an integral kernel. We then use approximation methods to study a pointwise nonlocal boundary condition depending on values of the solution at finitely many interior points. Particular attention is paid to the case in which the boundary condition depends on the value of the solution at a single interior point; the argument is then extended to the multipoint case.

Related existence results have previously been established mainly in spaces of continuous functions, with Krasnoselskii fixed point theorem in a cone as the principal tool. In contrast, the present approach treats solutions in the weak sense and relies on a hybrid fixed point argument involving the theory of matrices convergent to zero.

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Uniqueness results for nonlinear Dirichlet problems

Pasquale Candito

This talk deals with uniqueness results for nonlinear Dirichlet problems obtained using different tools. Specifically, the first part is devoted to Laplacian problem investigated by using fixed-point theorem [1], while the second one concerns with a possible singular p -Laplacian problem studied combining truncation and variational methods [2].

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Convergence Analysis of Systems with Fixed Point Structure

Loredana Dafinoiu, Mircea Sofonea and Bruno Vassallo

Let X and Y be two arbitrary sets, $P \subset X \times Y$ and $Q \subset X \times Y$. We consider the problem of finding a couple $(u, \eta) \in X \times Y$ such that $(u, \eta) \in P \times Q$. We prove that, under appropriate assumptions, the existence of the solution of this problem is equivalent to the existence of fixed points of some associated operators Λ and Θ . Moreover, the solution is unique if and only if the corresponding fixed points are unique. Then, we move to the framework of metric spaces X and Y and, under additional assumptions, we provide necessary and sufficient conditions which guarantee the convergence of an arbitrary sequence $\{(u_n, \eta_n)\} \subset X \times Y$ to the solution (u, η) . Our abstract results have many applications in the analysis of various nonlinear systems. We illustrate their use in some examples, proving existence and uniqueness results, together with convergence criteria.

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Global and semi-global monotone solutions of advanced nonlinear differential equations with state-dependent arguments

Josef Diblík, Miroslava Ružicková and Gabriela Vážanová

We study advanced nonlinear differential equations with state-dependent arguments. To prove the existence of global or semiglobal monotone solutions, bounded from below and above by exponential-type functions, a monotone iterative method is used. The new findings are illustrated with examples. Previously known results are discussed, and several open problems are formulated.

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On quasilinear nonlocal problems of variational and non-variational types

Leszek Gasiński

We study a class of nonlocal quasilinear problems driven by the p -Laplacian-like operators of both variational and nonvariational types. The nonlocal term depends on the unknown function or its gradient. The existence of positive solutions as well as multiplicity and nonexistence results are established through a combination of variational methods, truncation techniques, set-valued analysis, and fixed-point arguments.

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Computer-Assisted Proof for Radial Solutions of Elliptic Equations with Coulomb and Hardy Potentials

Aleksander Ćwiszewski, Władysław Klinikowski and Jean-Phillipe Lessard

We are interested in radially symmetric solutions of an equation involving Coulomb and Hardy potentials. The problem is transformed to an ordinary differential equation on the half-line with asymptotic decay conditions. Following a constructive strategy, we divide the interval into three subintervals. On the first interval the solution is represented by a Taylor-type two-indices series, leading to recurrence relations for its coefficients. On the third interval the constructed solution is required to belong to the center-stable manifold generated by the first-order system derived from the equation; the Lyapunov-Perron operator is used to show the existence of such a manifold. To connect smoothly the Taylor-type solution and the part lying in the center-stable manifold, we use a Chebyshev series representation on an intermediate interval. This leads to a zero-finding problem for a map defined on a Banach space of coefficients. Zeros are obtained using a variant of the Newton-Kantorovich theorem and rigorous numerics based on interval arithmetic.

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Three solutions for elliptic Dirichlet boundary value problem with singular weight.

Piotr Kowalski and Joanna Piwnik

We consider a p -Laplacian Dirichlet boundary value problem with a Hardy-type singular weight on a bounded Lipschitz domain $\Omega \subseteq \mathbb{R}^n$ ($0 \in \Omega$, $2 \leq p < n$). Specifically, we seek weak solutions $u \in W_0^{1,p}(\Omega)$ of the problem

$$-\Delta_p u + \mu \frac{|u|^{p-2}u}{|x|^p} = \lambda f(u) + \gamma g(u) \quad \text{in } \Omega, \quad u|_{\partial\Omega} = 0,$$

where $\mu > 0$ and $\lambda, \gamma \geq 0$ are parameters. We assume that $f: \mathbb{R} \rightarrow \mathbb{R}$ is continuous, satisfies sublinearity conditions at zero and at infinity, and that its primitive $F(t) = \int_0^t f(s) ds$ attains positive values. The function $g: \mathbb{R} \rightarrow \mathbb{R}$ is assumed to have subcritical growth. Under these assumptions, applying the Ricceri three critical points theorem together with the Hardy inequality and the Browder–Minty theorem for uniformly monotone operators, we prove the existence of at least three weak solutions (two of them non-trivial) for λ belonging to some compact interval $[a, b] \subset (\beta, +\infty)$ and sufficiently small $\gamma \geq 0$.

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Buckling of a Gao Beam under Combined Axial and Vertical Loading

Jitka Machalová and Horymír Netuka

We investigate the buckling and post-buckling behaviour of the nonlinear Gao beam subjected to combined axial and vertical loading. The model introduced by Gao [1] extends the classical Euler–Bernoulli beam theory by incorporating a nonlinear term that makes it possible to describe moderately large deflections and nonconvex post-buckling regimes. The problem is formulated variationally through the corresponding potential energy functional, whose convexity properties determine the number and character of equilibrium solutions.

The main attention is paid to the influence of a prescribed vertical load on the critical axial load. For zero vertical loading, the critical value coincides with the classical Euler buckling load, in agreement with the purely axial case studied in [3]. For nonzero vertical loading, however, the critical axial load increases and the symmetric pitchfork bifurcation changes into an imperfect pitchfork bifurcation [2]. We describe critical curves separating parameter regions with a unique equilibrium solution from regions with three equilibrium solutions. These branches correspond to a global minimum, a local minimum and a saddle point of the potential energy functional.

The theoretical observations are complemented by numerical experiments for cantilever and simply supported beams under uniform and point vertical loads. We also discuss how material and geometric parameters, including Young's modulus, Poisson's ratio, beam thickness, width and length, affect critical loads, maximum deflections and energy values. The results contribute to a clearer understanding of the post-buckling response of Gao beams under realistic combined loading conditions.

Keywords: Gao beam; buckling; post-buckling analysis; combined loading; nonlinear beam model; imperfect bifurcation; variational formulation.

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Singular elliptic equations with $(p(z), q(z))$ -growth

Witold Majdak and Nikolaos S. Papageorgiou

We consider the following singular Dirichlet problem driven by a variable unbalanced growth differential operator and a singular reaction with an unbounded coefficient:

$$-\Delta_{p(z)}^\alpha u(z) - \Delta_{q(z)} u(z) = \frac{f(z)}{u(z)^{\eta(z)}} \quad \text{in } \Omega, \quad u|_{\partial\Omega} = 0, \quad u > 0. \quad (1)$$

The exponent $\eta \in C(\overline{\Omega})$ is assumed to satisfy $\eta(z) < 1$ only near $\partial\Omega$. We prove that this problem admits a unique bounded solution. Our talk is based on the results presented in the recently published article [1].

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Korteweg-de Vries equation perturbed by a random force

Elżbieta Motyl

The well-known Korteweg-de Vries equation describes propagation of shallow water waves that are weakly and nonlinearly interacting. The deterministic KdV equation without external forcing, i.e.

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + 6u \frac{\partial u}{\partial x} = 0, \quad x \in \mathbb{R}, \quad t > 0,$$

has been studied by many authors. In many situations, for example, while studying flow over a flat horizontal bottom with localized inhomogeneities or response in a channel to a surface disturbance, a forcing term has to be taken into account. Besides, KdV equation with random forcing has arisen in plasma physics. In the present talk we consider the initial-value problem for KdV equation perturbed by a Poisson random measure and prove the existence of a weak solution. The construction of the solution is based on the parabolic regularization technique, compactness method and the martingale representation theorem.

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Generalized History-Dependent Operators and Their Applications

Zhenhai Liu, Anna Ochal and Jing Zhao

We establish a unified fixed point theorem for a class of almost generalized history-dependent operators, extending the classical history-dependent operators by allowing the kernel to be integrable rather than constant. These operators provide a flexible framework for modeling memory effects in differential and integro-differential equations, encompassing Volterra and fractional integral operators as special cases. The framework developed here has broad applications in viscoelasticity, population dynamics with hereditary effects, and contact mechanics. Its main advantage is that it offers a more flexible tool for analyzing evolution equations with memory.

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Existence and Asymptotic Equivalence of Solutions to Nonlinear Volterra-Type Difference Equations

Urszula Ostaszewska, Ewa Schmeidel, Barbara Łupińska and Małgorzata Zdanowicz

We consider a Volterra-type difference equation involving a general multiplicative coefficient in the principal linear part. We present sufficient conditions for the existence of solutions with prescribed asymptotic behavior. In particular, we show that solutions of the nonlinear equation admit asymptotic representations in terms of solutions of the corresponding linear equation, implying that the nonlinear Volterra term is asymptotically negligible. The obtained results extend earlier contributions on the asymptotic theory of nonlinear difference equations to a broader class of Volterra-type equations with a general multiplicative coefficient in the principal linear part. The presented thesis are illustrated by representative examples.

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Self-similar solutions as large-time asymptotics for space-fractional Stefan problems

Sabrina Roscani and Piotr Rybka

We investigate the large-time behavior of the one-phase fractional Stefan problem with a Caputo flux defined by

$$q(x, t) = -k_\alpha {}^C D^\alpha u(x, t) = {}_0 I^{1-\alpha} [u(\cdot, t)](x) = \frac{1}{\Gamma(1-\alpha)} \int_0^x u_x(z, t) (x-z)^{-\alpha} dz.$$

Motivated by the general principle that the long-time behavior of evolution problems is governed by steady states or special solutions such as self-similar solutions, we prove that the solution converges, as $t \rightarrow \infty$, to the self-similar solution obtained in [2] for the one-phase space-fractional Stefan problem with a fractional Neumann-type boundary condition at the fixed face,

$$-{}_0^C D_x^\alpha u(0, t) = \frac{h_0}{t^{\alpha/(1+\alpha)}}.$$

The present work extends the corresponding result for the classical Stefan problem established in [1], where a variational approach was developed to analyze the large-time asymptotic behavior.

The fractional extension presented here not only demonstrates that the variational technique remains applicable in the nonlocal setting, but also incorporates several recent advances on Stefan problems with Caputo fluxes, including comparison and maximum principles, monotonicity of the free boundary, and regularity results in fractional Sobolev spaces.

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Properties of Summation Equations of Volterra Type Involving the p -Laplacian

Urszula Ostaszewska, Małgorzata Zdanowicz and Ewa Schmeidel

We establish sufficient conditions for the existence of solutions to a Volterra-type difference equation involving the p -Laplacian operator. Specifically, we investigate solutions that are asymptotically equivalent to a given sequence, depending on the coefficients of the neutral term. The main results are obtained by employing Schauder fixed-point theorem, covering both cases where the neutral coefficient is nonzero and the non-neutral case.

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MS3

Asymptotic methods in solid and fluid mechanics

On the Justification of a Contact Problem for Generalized Membrane Shells

Ángel Arós

We consider a three-dimensional model for generalized linear elastic membrane shells in contact with a deformable obstacle and we study the convergence of the solution of this model towards the solutions of the corresponding models for two-dimensional elastic generalized membrane shells when the small parameter of the model (thickness) tends to zero, all depending on the type of boundary conditions, curvature of the middle surface and description of the applied forces. We also show the convergence to the solution of the corresponding Koiter shells models under these conditions.

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Asymptotic Analysis of Adhesive Contact in Elastic Membrane Shells

C. Fernandes and Á. Arós

We consider a family of linearly elastic shells, all sharing the same middle surface, with thickness 2ε , clamped along their entire lateral face, which, upon deformation, may enter into adhesive contact with a foundation along their lower face. As a consequence of adhesion, interfacial bonding forces develop, influencing the mechanical response of the shell and the evolution of the contact interface.

We focus on the case of an elliptic membrane, for which we show that the solution pair $(\mathbf{u}(\varepsilon), \beta(\varepsilon))$, consisting of the displacement field, $\mathbf{u}(\varepsilon)$, and the bonding function, $\beta(\varepsilon)$, of the three-dimensional scaled variational contact problem, converges to a pair of limit functions $(\mathbf{u}, \beta)$ as $\varepsilon \rightarrow 0$. This limit pair can be identified with the solution of a two-dimensional variational problem, since $\mathbf{u} = (u_i)$ is independent of the transverse variable x_3 . In addition, our numerical analysis of the adhesive contact model provides a priori error estimates for the displacement field, the stress tensor, and the adhesion variable. We implement a fully discrete numerical scheme in the FreeFEM++ programming environment. A series of numerical simulations are conducted to demonstrate the performance and robustness of the proposed scheme.

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A new light on the interface condition between the flow in a porous medium and the free flow

Eduard Marušić-Paloka

In this paper, we derive a new effective interface condition governing the transition between porous and free flow regions of a fluid domain via asymptotic analysis. The proposed non-standard condition represents a Darcy-type law acting across the imaginary interface, asserting that the trace of the free-flow velocity is proportional to the difference in stresses on both sides of the interface.

Higher-order asymptotics reveals that the leading-order approximation corresponds to a no-slip condition, the first-order approximation to a non-penetration condition with tangential slip, whereas the second-order approximation acknowledges leaking across the interface. This hierarchical behavior is particularly relevant in modeling blood flow in the arteries, where the arterial wall behaves as a porous medium, allowing slow blood seepage relative to the main flow.

Keywords: porous medium, free flow, interface condition, macroscopic model.

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Mathematically rigorous justification of the Reynolds equation for a spherical bearing

Guy Bayada, José M. Rodríguez, Raquel Taboada-Vázquez

We prove that the solution of the Stokes problem in a domain between two spheres converges, as the separation between them tends to zero, to the solution (unique up to the addition of a constant) of the following Reynolds problem:

$$\frac{h^2}{12\mu} \frac{\partial p}{\partial \varphi} = -Rk_0 \text{ on } \varphi = \varphi_0, \quad (1)$$

$$\frac{h^2}{12\mu} \frac{\partial p}{\partial \varphi} = -Rk_1 \text{ on } \varphi = \varphi_1, \quad (2)$$

$$\frac{\partial}{\partial \varphi} \left(\frac{h^3 \sin \varphi}{12\mu} \frac{\partial p}{\partial \varphi} \right) + \frac{\partial}{\partial \theta} \left(\frac{h^3}{12\mu \sin \varphi} \frac{\partial p}{\partial \theta} \right) = \frac{R^2 \omega \sin \varphi}{2} \frac{\partial h}{\partial \theta} \quad (3)$$

where

$$\vec{X}(\varphi, \theta) = R(\sin \varphi \cos \theta, \sin \varphi \sin \theta, \cos \varphi), \quad (4)$$

$$\varphi \in (\varphi_0, \varphi_1), \quad 0 < \varphi_0 < \varphi_1 < \pi, \quad \theta \in (0, 2\pi), \quad (5)$$

is the parameterization of the inner sphere, R is its radius, $p(\varphi, \theta)$ is the pressure, $h(\varphi, \theta)$ gives the distance between the two spheres, $k_0(\theta)$ and $k_1(\theta)$ are the averaged input/output flows at $\varphi = \varphi_0$ and $\varphi = \varphi_1$, respectively, ω is the angular velocity of the inner sphere (we assume the outer sphere is stationary) and μ is the dynamic viscosity.

To our knowledge, this is the first mathematically rigorous justification of the Reynolds equation for a spherical bearing.

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Asymptotic Analysis of Linearly Elastic Flexural Shells in Normal Compliance Contact

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We consider linearly elastic flexural shells in frictionless normal compliance contact with a deformable foundation. Starting from the three-dimensional elasticity problem, asymptotic analysis leads to a two-dimensional flexural shell contact model, for which convergence of the three-dimensional solutions is established. Since this limit problem is posed on the space of inextensional displacements, making its numerical approximation challenging, we introduce a Koiter-type contact model defined on a larger admissible space. We prove that this model converges to the flexural shell problem as the penalization parameter tends to zero, and present a penalized formulation together with numerical simulations illustrating the performance of the proposed approximation.

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MS4

**Energy harvesting, smart self-power
sensors, machine structure dynamics and
diagnostics**

Performance Evaluation of YOLOv26 Architectures in Rotary Tedder Tooth Damage Detection

Kamil Felter, Arkadiusz Mystkowski

The automated detection of mechanical defects in agricultural machinery is essential for the efficiency of modern smart farming [1,2]. Building upon our previous research, this study advances the inspection of rotary tedder teeth by evaluating the latest YOLOv26 architectures. We utilized a high-performance dataset, previously optimized for the YOLOv11x model, to train and benchmark both the YOLOv26s and YOLOv26m variants [3,4]. Our evaluation, comprising mAP@50, mAP@50-95, confusion matrices, and training efficiency, confirms the superiority of this newer architecture. The YOLOv26s model achieved a mAP@50-95 of 0.567, outperforming the YOLOv26m variant (0.553) by 2.53% and the YOLOv11x baseline (0.562) by 0.89%. These results demonstrate that the YOLOv26 family not only ensures detection precision but also offers a robust, efficient solution for real-time diagnostics in agricultural edge computing environments, marking a significant first step toward a fully realized computer vision-based damage detection system.

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Autonomous agriculture machine health monitoring systems using convolutional neural networks optimized by genetic

Arkadiusz Mystkowski, Mateusz Sewiolo, Mircea Sofonea

This work presents the complete design, implementation, and validation of a diagnostic system for a rotary tedder integrating state-of-the-art sensing, communication, and deep learning methodologies. The core innovation lies in a robust fault detection framework utilizing Convolutional Neural Networks (CNNs), whose architecture is automatically optimized through a Genetic Algorithm (GA), significantly enhancing diagnostic accuracy and adaptability.

A dedicated on-board monitoring unit was engineered, comprising a network of 12 temperature sensors, 5 vibration sensors, and rotation sensors strategically installed on critical driveline components. Data acquisition is managed by an STM32 microcontroller platform, which handles sensor interfacing, preliminary signal processing, and GPS-tagged data logging to an SD card. For real-time telemetry, the unit incorporates GPRS connectivity, transmitting data via the MQTT protocol to a cloud-based platform. This infrastructure enables both online monitoring and offline analysis of machine states.

The core diagnostic intelligence employs a hierarchical analytical methodology. Initial health indicators are derived from time-domain vibration analysis, including RMS and Crest Factor. For advanced fault detection, the system utilizes deep learning. The primary innovation lies in the application of a custom Genetic Algorithm to automate the design and optimization of CNN architectures for fault classification. The GA optimizes the number and type of parallel input pathways, processing raw signals, FFT vectors, spectrograms, and analysis vectors, as well as the configuration of convolutional layers and the structure of dense classification layers. This approach eliminates suboptimal manual architecture tuning, ensuring a model tailored to the specific characteristics of agricultural machine vibration signatures.

The system was rigorously validated under laboratory and field conditions using SaMASZ P8-890 and Z2-1000 machines. Trained and optimized on data representing healthy operation, gear tooth faults, and lubrication shortages, an initial single-input CNN achieved 99–100% accuracy in distinguishing these fundamental states. A more complex, evolved Multiple-Input, Multiple-Type Parallel CNN demonstrated superior performance in diagnosing nuanced, multi-class fault scenarios. This final architecture achieved a mean diagnostic accuracy of 92.67% across nine distinct health conditions, including varying severities of mechanical faults.

Keywords: Machine Fault Diagnosis, Convolutional Neural Networks, Genetic Algorithm Optimization, Predictive Maintenance, Smart Agriculture.

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Proposal of Adaptive Computed Torque Controller for Manipulators under Payload Uncertainties

Arkadiusz Nieciecki, Arkadiusz Mystkowski

The rapid development of modern industries requires robotisation that extends beyond simple tasks, driving a growing demand for control systems capable of adapting to dynamic changes in their environment and manipulated objects [1-3]. To address this, the proposed Adaptive Computed Torque Control (ACTC) architecture for a manipulator under payload uncertainties integrates a classical CTC with two parallel backpropagation neural networks. A BPNN Tuner dynamically adjusts the PD controller gains, while a BPNN Compensator counteracts unmodelled dynamics in real time. Compared to the classical CTC baseline, the ACTC significantly improves tracking precision under sudden payload variations, reduces the Joint 1 RMSE to 0.108 rad, and achieves a 10-fold reduction in the Joint 2 RMSE to 0.128 rad, while drastically dropping the Joint 2 IAE from 99.080 to 8.223 rad-s.

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Analysis of a bluff body geometry through vibrations induced by low-speed air vortices while harvesting energy using a piezoelectric cantilever

Arkadiusz Mystkowski, Piotr Wolszczak, Sreeja Sadasivan and Grzegorz Litak

This article discusses experiments on energy harvesting from wind-induced vibrations (WIVEH), using systems consisting of a flexible beam with piezoelectric transducers (PZTs) coupled to an irregular solid geometry. The study examined streamlined solid structures mounted singly and in pairs on the beam. Characteristics of vibration, frequency, and output voltage were obtained at air speeds ranging from 3.5 to 15 m/s.

The importance of nonlinear aerodynamic excitation mechanisms of vortex-induced vibrations (VIV), including galloping and flutter effects, was discussed. The obtained results demonstrate the influence of irregularly shaped solids on the beam's vibration amplitude, vibration frequency, and energy harvesting efficiency. Among the paired solid configurations, the two-cylinder configuration achieved the highest vibration amplitudes and an 89% increase. However, the single-cylinder configuration provided a moderate and predictable voltage increase, approximately 75 mW/cm³. Experimental observations have proven that two-cylinder configurations can be effective in low, constant airspeed conditions, and a cylinder-cuboid hybrid can be dedicated to broadband, variable-speed environments.

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RBF-Based Adaptive Control Concept for a 6-DOF Flexible Manipulator: A Simulation Study on Precision and Energy Efficiency

Mateusz Zalewski, Arkadiusz Mystkowski

Modern lightweight manipulators suffer from flexibility-induced resonant oscillations[1,2,3]. This paper proposes an RBF-based Adaptive PD control architecture for a theoretical 6-DOF flexible-link manipulator modeled with a 0.55 Hz dominant resonance. To suppress vibrations, an RBF neural network dynamically adjusts baseline PD gains in real-time using pure online Lyapunov-based learning. Verified entirely within a rigorous sampled-data simulation framework, the proposed method improves trajectory tracking precision by 72.5% (RMS error reduction) over a classical PD controller. Crucially, proactive vibration damping reduces total mechanical energy consumption by over 25%.

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MS5

Uncertainty quantification and microstructural disorder

Mechanical Response and Damage Evolution of 3D-Printed Engineered Cementitious Composites Reinforced with SMA Fibres under Cyclic Loading

Muhammad Faizan Ali, Qian Hui and Zongao Li

Additive manufacturing of cement-based materials provides new opportunities for developing sustainable and high-performance construction systems; however, mechanical anisotropy, interlayer weakness, and progressive damage accumulation remain critical challenges. This study investigates the mechanical response and damage evolution of 3D-printed engineered cementitious composites reinforced with Shape Memory Alloy fibres under cyclic flexural loading. The influence of printing-induced anisotropy and fibre reinforcement on load-deflection behaviour, stiffness degradation, energy dissipation, and crack evolution are experimentally evaluated. Digital image correlation is employed to monitor crack initiation, propagation, and recovery behaviour during cyclic loading. The results demonstrate that fibre reinforcement improves crack control capability and enhances the postcracking mechanical stability of printed ECC. The findings provide insights into the damage mechanics and structural performance of advanced 3D-printed cementitious composites for future resilient construction applications.

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On an isotropic porous solid cylinder: the sensitivity analysis of the pressure, stresses, and displacement

Hadi Asghari

Carrying out a sensitivity analysis as an uncertainty and variance-based technique clarifies the effects of material input parameters on the pressure, stresses, and displacement of an isotropic porous solid cylinder, which models hard mechanical systems like bone. This presentation considers the modeling of the system through the governing equations of Biot's poroelasticity in cylindrical polar coordinates, where solutions are obtained by applying radial boundary conditions. The sensitivity analysis is performed on the solutions for pressure, stress components, and displacement, utilizing ranges of parameters that reflect the characteristics of bone. The research indicates that the time t^* has the most considerable impact on pressure, stress components, and displacements. Furthermore, it is found that the Poisson ratio ν has a more significant influence on the pressure response than the shear modulus μ , while the shear modulus μ is more critical than other parameters in relation to radial and circumferential stresses. There are significant interactions between Biot's coefficient α , the Poisson ratio ν , the non-dimensional radius R^* of the bone, and Biot's modulus M when examining interstitial pressure, which is vital for bone remodeling and fracture healing. This study lays the groundwork for a deeper understanding of the interactions among all parameters necessary to accurately capture the behavior of hard mechanical systems, such as bone and its potential for remodeling.

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Reliability response surfaces for a nonlinear deflection problem under geometric uncertainty

Rafał Błoński, Marcin Kamiński

This work presents a reliability-based analysis of a nonlinear deflection problem under geometric uncertainty. The investigated system is a cantilever column subjected to an axial compressive force and a transverse distributed load. The deterministic response is described by an Euler-Bernoulli beam formulation including the P-Delta effect, where the axial force acting on the laterally deflected configuration increases the final displacement. The column length is modelled as a normally distributed random variable, and its coefficient of variation defines the level of geometric uncertainty. A deflection-based limit state function is defined as the difference between the allowable deflection and the nonlinear P-Delta deflection response.

The reliability index is evaluated as a function of the coefficient of variation of the column length and the axial load multiplier. Three approaches are compared: Monte Carlo simulation, an analytical approximation based on a truncated series expansion, and the tenth-order stochastic perturbation method. The calculated reliability indices are then approximated by bivariate polynomial response surfaces using the least-squares method. These surfaces are used to interpret reliability degradation and to draw reference reliability contours.

The results show that increasing geometric uncertainty and axial load level reduce reliability. The response surface representation provides a compact framework for interpreting the analysed nonlinear reliability problem.

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Data-Driven Uncertainty Quantification of Mass Transport Properties in Polymer Composites

Mustafa Okumus and Lukasz Figiel

This work develops a fully Bayesian computational framework for inferring in-situ mass transport characteristics (permeability) of the main constituents (fibre, matrix and interface) of polymer composites. The framework combines experimental permeation measurements with stochastic three-dimensional representative volume element (RVE) simulations and Gaussian process (GP) surrogate modelling. It is applied to a case study of hydrogen permeation in a MTC510 carbon fibre/epoxy system. The inference framework utilises a limited set of new hydrogen permeation measurements on the system and experimentally-informed priors on the matrix permeability (ex-situ). Constituent permeabilities and GP hyperparameters are inferred jointly, extending the Bayesian uncertainty propagation to include GP hyperparameter uncertainty alongside uncertainties arising from experimental observations, stochastic microstructural variability and surrogate approximation. Posterior predictive checks demonstrate that the inferred constituent permeabilities reproduce the experimentally measured composite overall permeability. The posterior distributions indicate a substantially lower interphase permeability than the surrounding epoxy matrix. Transferability is further demonstrated using literature data for a carbon fibre-reinforced PA6 composite, revealing differences in the inferred matrix-interphase permeability relationship between thermoset and thermoplastic systems. The framework provides a general approach for identifying experimentally inaccessible constituent transport properties in composite materials.

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Mechanical response of the cellular material with incorporation of Fiber

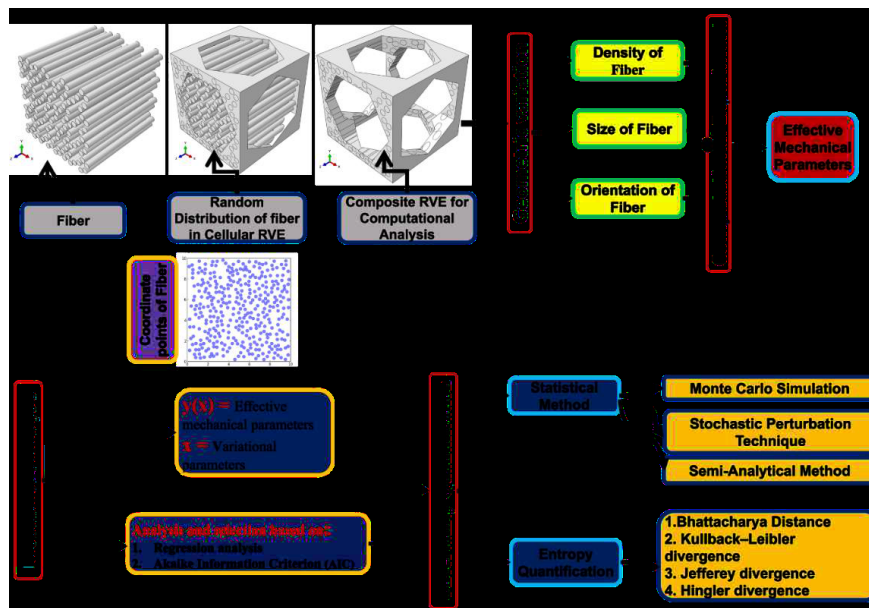
Safdar Iqbal and Marcin Kaminski

The mechanical performance of cellular materials can be significantly improved by incorporating a reinforcing material to form a composite cellular structure. In this study, fibers were embedded within the representative volume element (RVE) of a cellular material to develop a fiber-reinforced composite architecture. A numerical homogenization approach based on the finite element method was employed to determine the effective mechanical parameters of the composite cellular material.

To evaluate the influence of uncertainties in material properties and structural response, a polynomial response model was developed and used for uncertainty propagation. Three probabilistic methods: Monte Carlo simulation, the stochastic perturbation technique, and the semi-analytical method were implemented and compared in terms of their ability to predict the probabilistic characteristics of the effective mechanical response. Furthermore, information-theoretic measures, including the Bhattacharyya distance, Kullback–Leibler divergence, Jeffrey’s divergence, and Hellinger distance, were employed to quantify the similarity and divergence among the probability distributions obtained from the different probabilistic approaches.

The proposed computational framework provides an efficient and reliable methodology for predicting the effective mechanical properties and quantifying the associated uncertainties of fiber-reinforced composite cellular materials, thereby contributing to the design and reliability assessment of advanced lightweight engineering structures.

Keywords: Cellular material; fibers; numerical analysis; mechanical property; probabilistic analysis.



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Exponential polynomial closure method for dynamical systems under renewal impulse process excitation

Radosław Iwankiewicz and Anna Jabłonka

The dynamical system subjected to a renewal impulse process excitation is considered. An Erlang process is assumed as a process driving the impulse process excitation. For such a problem an auxiliary jump process characterized by a Markov chain is introduced and consequently the joint probability density of the state vector and of the underlying Markov states is governed by the coupled set of integro-differential equations [1]. The number of these equations equals the number of Markov states of the auxiliary jump process. This is the generalization of the integro-differential Kolmogorov-Feller equation governing the probability density of the state vector of the dynamical system under a Poisson impulse process excitation. An approximate exponential polynomial closure method is developed for this problem. This method was applied e.g. for non-linear systems under Gaussian white noise excitations [2] and under a Poisson impulse process excitation [3]. The approximate form of the response probability density function is assumed as the exponential function of the polynomial of state variables with time-varying coefficients. The approximate probability density function is inserted into the governing integro-differential which results in the residual functions. It is required that the residuals be orthogonal with respect to suitably chosen trial functions. The result is the set of ordinary differential equations governing the coefficients of the underlying polynomial, which has to be solved numerically. As an example a linear oscillator is considered.

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Sensitivity and Uncertainty Quantification of Dynamic Characteristics of Multilayered Viscoelastic Beams with Fractional Zener Damping

Marcin Kamiński and Maciej Przychodzki

The paper presents an uncertainty quantification and global sensitivity analysis of the dynamic characteristics of multilayered viscoelastic beams. The considered structure consists of elastic and viscoelastic layers arranged symmetrically with respect to the mid-plane. The mechanical behavior of the multilayer beam is described using a refined zig-zag theory, while the viscoelastic layers are modeled by a fractional Zener constitutive law. The dynamic response is obtained from a nonlinear complex eigenvalue problem formulated in the Laplace domain. The analysis is focused on the modal damping factor, the natural frequency, and the dimensionless frequency-relaxation parameter. The uncertain input parameters include the relaxed and non-relaxed elastic moduli of the viscoelastic material, the relaxation time, the fractional derivative order, and selected layer thicknesses. Latin Hypercube Sampling is used to generate input realizations, whereas polynomial surrogate models are constructed to reduce the computational cost of repeated eigenvalue analyses. First-order and total-order Sobol indices are then evaluated to identify the dominant sources of output variability. The results show that the sensitivity mechanisms governing modal damping and natural frequency are fundamentally different. The modal damping factor is mainly controlled by viscoelastic material parameters and relaxation-related effects, although the dominant parameter depends strongly on the material. In contrast, the natural frequency becomes highly sensitive to geometric uncertainty, especially to the thickness of the viscoelastic layer. The study also demonstrates that Sobol rankings for damping may depend on the assumed uncertainty bounds, whereas the rankings for natural frequency are more stable. These findings indicate that reliable design of multilayered viscoelastic beams requires material-specific uncertainty analysis rather than universal sensitivity rankings.

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Moment Propagation versus Design-Point Search: The Stochastic Perturbation Method Convergence in Tunnel Reliability

Nadir Albo and Marcin Kamiński

The stochastic perturbation method is applied to the reliability assessment of tunnel convergence, a problem so far addressed almost exclusively by design-point search techniques. The mechanical model is the classical plane-strain solution for a circular opening in a Mohr-Coulomb elastic-perfectly plastic medium under hydrostatic in situ stress. Because the response is explicit, the perturbation expansion is carried out with exact symbolic derivatives, and the first two probabilistic moments of the convergence are obtained at negligible cost through a mixed second-order-mean and first-order-variance scheme. The resulting moments feed a Cornell reliability index, which is compared with first-order reliability method results for the same limit state and operating point. The Monte Carlo simulation confirms the stochastic perturbation method moments, thereby identifying the discrepancy in the index rather than in the propagation. Both approaches agree on the classification of input variables based on sensitivity.

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Shannon Entropy as a Universal Metric for Uncertainty Quantification of Transport Phenomena and Phase Transitions in Computational Mechanics

Marcin Kamiński, Rafał L. Ossowski

This work demonstrates the efficacy of Shannon entropy as a universal, information-theoretic metric for quantifying uncertainty propagation across diverse physical phenomena in computational mechanics. Unlike traditional stochastic approaches that rely on decoupled, high-order statistical moments, probabilistic entropy compresses total system disorder into a single scalar field, offering critical insight into deterministic stability under input scattering. We synthesize three physical regimes from recent research: stochastic fluid hydrodynamics, reaction-diffusion kinetics, and microstructural parameter homogenization. The results show that entropy identifies regions and thresholds of increased thermal-hydraulic randomness, including nanoparticle clustering and percolation transitions, where minor volumetric fraction uncertainties cause disproportionate variations in effective properties. Ultimately, this work establishes Shannon entropy as a powerful diagnostic tool for the structural design of complex transport processes in uncertain physical environments.

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Probabilistic strength analysis of UDFRP plate under transverse tensile loading using random field modeling considering random microscopic geometry and material properties

Sei-ichiro Sakata, George Stefanou, Shintaro Hokamura, and Yu Nakayama

Multiscale probabilistic strength analysis of random composites becomes an important topic for quantitative strength estimation and robust design of composite structures. Several papers on multiscale probabilistic analysis considering microscopic randomness and the resulting random spatial distribution of local apparent mechanical properties have been published, e.g. [1][2]. In this framework, the probabilistic analysis of local apparent mechanical properties, such as the elastic modulus and strength, and random field modeling technique play an important role. In particular, for this purpose, strength analysis is performed with the random field modeling referring to an actual SEM image of a specimen, and the effectiveness of the approach for probabilistic strength evaluation of composites was discussed. In these reports, the randomness in fiber arrangement is taken into account as input. On the other hand, the detailed influence of other random factors, such as material properties, has not been investigated in detail. In this work, microscopic material properties such as the microscopic fracture stress of resin and interface are also considered as random inputs, and probabilistic strength analysis of a randomly arranged unidirectional fiber reinforced (UDFRP) composite plate under a transverse tensile loading is attempted. Based on the numerical results, the influence of the microscopic uncertainty in those properties on the probabilistic properties of the apparent transverse strength of the randomly arranged UDFRP composite plate is investigated, and their importance for the probabilistic strength analysis of composite structures will be discussed.

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Development of a simplified analytical model for dynamic analysis of lattice towers with variable cross-section

Hanna Weber and Radosław Iwankiewicz

Due to their slenderness, guyed lattice towers are structures sensitive to dynamic loads. Earthquakes can excite both the structure and the cables, even if the seismic load source is located at a significant distance from the system. Ground vibrations lead to bending deformations of the tower, which in turn constitute a second source of excitation of the cables at their anchoring points, in addition to the base motion excitation at supports. The response of the guy line to the seismic loads can be an important tool used during the design process, considering maximum values of displacements and fatigue of the material. In the presented approach, a simplified analytical model of a variable-cross-section lattice tower with one or three guy lines is proposed, where the structure is simplified to a cantilever beam with an equivalent constant cross-section and mass per unit length. The earthquake is modeled as a filtered Gaussian white noise process, which is characterized by Kanai-Tajimi spectrum [1]. The Ritz method and Lagrange's equation is used to obtain the set of nonlinear differential equations of motion. A detailed discussion is given of the method for determining the equivalent characteristics of the simplified tower model based on the formulas derived from literature [2,3] and their modifications. The obtained results are compared with those obtained from numerical models of lattice towers with various geometry with one or three guy lines. The proposed approach can be used to create an application or software that allows for quick determination of basic model parameters, such as the equivalent moment of inertia and mass per unit length or the fundamental frequency of a system with one or three guy lines, without the need to create a complex computational model of the system. The obtained equivalent values for the structure can be then used to determine the cable response under stochastic seismic excitation.

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MS6

Numerical Analysis and Computational Methods for Mechanics

Coupled flow and mechanics in a fractured porous medium

Kundan Kumar

We present a mixed dimensional model for a fractured poro-elastic medium including contact mechanics. The fracture is a lower dimensional surface embedded in a bulk poro-elastic matrix. The flow equation on the fracture is a Darcy-type model that follows the cubic law for permeability, while the bulk poro-elasticity is governed by fully dynamic Biot equations.

The fracture flow is coupled to bulk flow and geomechanics. Contact conditions are expressed through normal compliance and Coulomb friction. The resulting variational problem consists of a linear partial differential equation coupled to a nonlinear variational inequality. We establish well-posedness and provide a contractive iterative scheme. We also propose and analyze a fully discrete finite element scheme with implicit Euler time integration, proving existence, uniqueness, stability, and a priori error estimates. Numerical results are compared with the theoretical estimates. This is joint work with Maarten V. de Hoop, Marius Nevland, Inga Berre, Jakub Wiktor Both, and Eirik Keilegavlen.

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Numerical treatment of dynamic contact problems with the Damped Normal Compliance condition

Piotr Bartman-Szwarc

In this talk, I will discuss numerical approaches to dynamic contact problems formulated with the Damped Normal Compliance (DNC) condition. The DNC condition combines the normal compliance law with a damped normal response, so that the reactive obstacle possesses both stiffness and damping, and energy is dissipated upon contact. This provides a more accurate description of the contact process and makes the classical coefficient of restitution redundant. I will describe a finite element scheme for the resulting discrete problems. I will also comment on how the associated discrete problems can be treated within an optimization-based framework, which opens the way to handling more general, possibly nonsmooth, contact laws.

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Guaranteed Bounds for the Critical Buckling Load of the Euler–Bernoulli and Gao Beam Models

Jana Burkotová, Jitka Machalová and Tomáš Vejchodský

This contribution presents a computational framework for obtaining guaranteed bounds for the critical buckling load at which instability occurs in slender structures. The buckling analysis is formulated as an eigenvalue problem, where the smallest eigenvalue corresponds to the critical load. Reliable computation of this eigenvalue is thus essential for the safe design and assessment of engineering structures. While finite element approximations provide upper bounds for the eigenvalues through the min–max principle, obtaining lower bounds is more challenging. To derive guaranteed lower bounds, the proposed approach [1] combines finite element approximations with interpolation error estimates and a Wirtinger-type inequality. The method is first developed for the classical Euler-Bernoulli beam with piecewise constant bending stiffness and is subsequently extended to beams with variable bending stiffness. Finally, the approach is applied to the nonlinear Gao beam model by exploiting the equivalence of its first eigenvalue with that of the linear Euler-Bernoulli problem [2]. Several numerical examples are presented to demonstrate the accuracy and convergence rates of the proposed method.

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The Boundary Element Method in Deterministic and Probabilistic Approaches for Surface Girder Mechanics Problems

Michał Guminiak and Marcin Kamiński

The paper presents the application of the Boundary Element Method (BEM) to the analysis of statics, dynamics and stability of surface girders modeled according to the Kirchhoff-Love plate theory. Due to its specific nature, this method is not as widely used as the Finite Element Method (FEM). Nevertheless, the Boundary Element Method can be successfully applied to classical problems of solid mechanics, as well as to more complex problems concerning the interaction of engineering structures with fluids, such as shells interacting with air and plates interacting with liquid. The BEM is particularly well suited to this type of problem. The BEM is based on the knowledge of a fundamental solution or a set of fundamental solutions. This is both an important advantage and a limitation of the method. The advantage consists in the possibility of calculating boundary integrals analytically, while the limitation is related to the possible loss of convergence of the solution at high levels of discretization. Another important aspect of BEM analysis is the formulation of boundary integral equations. In dynamic and initial stability analysis, these equations may often take the form of boundary-domain integral equations. They can be formulated in either a singular or nonsingular manner. A characteristic feature of BEM is the possibility of using constant-type elements, that is, elements with a constant distribution of the boundary value. Although seemingly simple, such elements are very effective in various engineering analyses, from statics through dynamics to plate stability. An interesting problem is the dynamics of coupled systems, for example two plates surrounded by a fluid. In such systems, vibrations are transmitted through the fluid medium, and it is essential to construct a complete, generally asymmetric fluid mass matrix. The BEM can be used for this purpose. In the case of an incompressible fluid, the construction of the fluid mass matrix is relatively simple if the domain is divided into constant subdomains. In the case of a compressible fluid, the solution, expressed in terms of the angular frequency of natural vibrations, is obtained in the set of complex numbers, and the fluid mass matrix becomes a complex matrix. Its real part represents the inertial forces of the fluid, while its imaginary part accounts for damping, especially the radiation of energy from the plate-fluid system. The deterministic formulation of a plate mechanics problem in the BEM approach may be treated as one component of a more general probabilistic approach to plate mechanics. According to this concept, one may select a design parameter, either geometric or physical, and solve a finite number of deterministic problems as a function of this parameter. Then, a probability density distribution for the occurrence of the selected design parameter is assumed. Most often, the Gaussian normal distribution is used. Since the probability density distribution is continuous, it is necessary to construct response fit curves on the basis of a finite number of known deterministic solutions. These curves usually take the form of third- or fourth-degree polynomials, whose coefficients may be calculated, for example, by the least squares method. The response fit curves allow one to determine random estimators in a semi-analytical approach. The most important of these are the expected value and the coefficient of variation, expressed as functions of the coefficient of variation of the argument, that is, the design parameter treated as an independent variable. In addition, one may determine the probabilistic entropy, expressed by probabilistic estimators such as the expected value and the standard deviation of the sought solution. This approach enables the estimation of reliability and safety measures of the structure.

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A Multilevel Graph Neural Network Framework for Model Reduction in Contact Mechanics

Michał Jureczka

The numerical simulation of dynamic contact problems in viscoelastic solids is computationally demanding, particularly when high spatial resolution is required to accurately capture localized contact phenomena. This work presents a data-driven model reduction strategy that combines the Finite Element Method (FEM) with Graph Neural Networks (GNNs) to approximate fine-scale solutions at a significantly lower computational cost. The proposed methodology employs a two-level discretization. The governing contact problem is solved on a coarse finite element mesh, while a graph neural network reconstructs the deformation of a corresponding fine mesh from the coarse-scale solution. To facilitate information exchange across discretization levels, we introduce a hierarchical graph representation with directed inter- and intra-mesh connections. The approach is developed for dynamic contact problems governed by a linear normal compliance law and Coulomb friction. A normalization procedure based on rigid-body translation and rotation is incorporated to preserve temporal coherence between coarse and fine mesh configurations throughout the simulation. The resulting framework is implemented in JAX and assessed by comparison with classical model reduction techniques, including Principal Component Analysis (PCA) and skinning-based interpolation methods. The presented approach demonstrates that graph-based learning can serve as an effective approximation for high-dimensional finite element solutions, offering a promising direction for model order reduction in computational contact mechanics and other multiscale problems arising in continuum mechanics.

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Thermo-hydro-mechanical analysis in the vicinity of tunnel excavation

Jaroslav Kruis, Tomáš Koudelka, Tomáš Krejčí

In deep radioactive waste repositories, special attention is paid to the excavation damage zone, which is formed during excavation in the vicinity of the tunnel. This is because high requirements are placed on the repository's impermeability [1]. This paper focuses on the coupled problems of heat transport-mechanics and heat and moisture transport-mechanics. These coupled problems are solved within the framework of a single disposal borehole [2]. Given the very long time scale under study (tens of thousands of years), simpler axially symmetric models are used initially. A three-dimensional numerical model is used to provide a more detailed and accurate description of the disposal boreholes.

Time integration is based on a generalized trapezoidal rule with an adaptive step size to achieve extremely short time steps. The finite element meshes contain hundreds of thousands of degrees of freedom and sometimes as many as millions of finite elements. Special attention is given to solving systems of algebraic equations. Both direct and iterative methods are used, particularly sparse direct solvers and, where possible, the conjugate gradient method. The results are compared.

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Adhesive rolling contact problem with elastoplasticity and hardening

Andrzej Myslinski

The paper is concerned with the analysis and numerical evaluation of adhesion in rolling contact problems. The contact phenomenon, between a rigid wheel and elasto-plastic rail lying on a rigid foundation, includes Coulomb friction, frictional heat generation as well as the wear of the contacting surfaces. The displacement and stress of the rail in contact are governed by the coupled elasto-plastic and heat conductive equations. The wear depth function appears as an internal variable in the non-penetration condition updating the gap between the worn surfaces of the bodies. The tangential and normal contact forces are updated due to adhesion described by RCC model. The existence of solutions to this contact problem is shown. The finite element framework is proposed to solve this contact problem numerically. The splitting approach is used to solve the original coupled thermo-elasto-plastic problem. The plastic flow and friction inequality conditions are reformulated as equality conditions using the nonlinear complementarity functions. Generalized Newton method is used to solve the discretized problem. The proposed approach is validated through a set of numerical examples.

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Inverse-free Newton's Method

Magdalena Nockowska-Rosiak

We present a modification of Newton's method for finding a zero of a multivariable function without an inverse of a matrix in a recurrence. We prove at least quadratic convergence of a Newton-type method avoiding matrix inversion under standard assumptions, and compare modified and classical Newton's methods numerically.

The talk is based on joint work with Marcin Massalski from Lodz University of Technology, [2].

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MS7

**Recent advances in non-linear and
non-smooth computational mechanics**

A unified semi-smooth Newton framework for large-strain SMA contact: application to stent deployment

Mikaël Barboteu, Francesco Bonaldi, Serge Dumont, Franck Jourdan, Rawane Mansour, Thach-Hoang Nguyen, and Vo Anh Thuong Nguyen

We propose a unified computational framework for large-strain dynamic contact in shape-memory-alloy (SMA) structures, with application to stent deployment in an artery-like environment. The model couples persistent unilateral contact and Coulomb friction at interfaces with finite-strain SMA inelastic transformations. A key feature is that both interface constraints and constitutive transformation conditions are written in a common KKT/NCP complementarity form. The resulting nonsmooth problem is solved by a single semi-smooth Newton–PDAS strategy, where Gauss-point active sets for SMA evolution are treated within the same loop as contact active sets. A midpoint time discretization is adopted to preserve robust transient behavior. The framework is assessed on academic superelastic and dynamic plasticity-type benchmarks, including exact-solution-oriented verification settings. It is then validated on a 2D stent–artery contact test under loading–unloading cycles with frictional interaction. Numerical results show competitive iteration counts versus radial-return-type updates. The proposed monolithic architecture provides a practical and scalable basis for advanced patient-oriented simulations with natural extensions toward 3D.

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MaMAL: Matrix-free Multigrid Augmented Lagrangian Method for Contact

Gabriele Marchi, Austen Nelson, Panayot Vassilevski, Rolf Krause and Patrick Zulian

High-performance computing is essential for efficiently solving large-scale contact problems. Simulating such phenomena at engineering scale is often limited by computational resources, making it crucial to design algorithms that fully exploit modern hardware such as multi-core CPUs and GPUs. Iterative solvers and preconditioners play a central role in this efficiency.

Monotone multigrid methods offer optimal complexity and robustness for variational inequalities arising in contact mechanics. In parallel, augmented Lagrangian formulations provide a flexible way to handle over-constrained and fuzzy constraints while retaining a clear variational structure.

We introduce MaMAL, a Matrix-free Multigrid Augmented Lagrangian method for large-scale contact problems. The method combines multilevel nonlinear smoothing, constraint-aware coarse operators, and augmented Lagrangian updates in a matrix-free framework designed for modern GPU architectures.

Our implementation uses matrix-free differential operators and memory-efficient semi-structured meshes to discretize elasticity equations. We present the MaMAL algorithm with emphasis on nonlinear smoothing, multiplier updates, constraint coarsening, and the interaction between augmentation parameters and multigrid convergence.

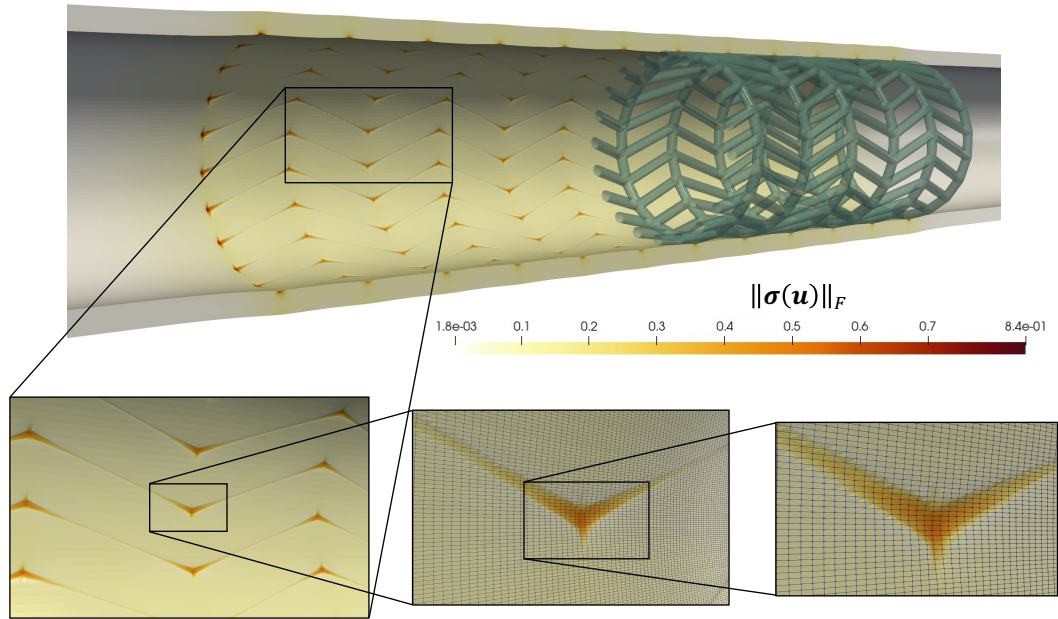


Figure 1: Stress distribution and local mesh resolution for a nonlinear contact computation.

We evaluate performance on the Grace-Hopper superchip of the CSCS Alps supercomputer. Emphasis is placed on single-node GPU performance, kernel design, and convergence behavior in representative contact scenarios. Finally, we demonstrate MaMAL's scalability on large-scale problems with hundreds of millions of degrees of freedom.

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Sufficient Conditions for Finite-Time and Fixed-time Stability of Time-Varying Differential Equations Using Dual Dynamic Programming

Radosław Matusik and Andrzej Nowakowski

In this work, we investigate the finite-time and fixed-time stability of impulsive differential equations by reformulating the stability analysis problem as an optimal control problem. This transformation provides a unified framework that enables the use of powerful tools from optimal control theory for the study of stability properties in impulsive dynamical systems. Building upon this formulation, we develop dual dynamic methodologies together with corresponding verification theorems, which allow us to derive rigorous and easily verifiable sufficient conditions ensuring both finite-time and fixed-time stability.

A central contribution of the proposed approach is the explicit treatment of impulsive effects as control variables within the optimization framework. Unlike conventional methods, where impulses are typically regarded as predefined system components or external perturbations, our formulation incorporates them directly into the control structure. This perspective not only broadens the analytical framework but also provides greater flexibility in characterizing and influencing system behavior. At the same time, the inclusion of impulses as decision variables introduces additional mathematical challenges, since the interaction between continuous dynamics and discontinuous state jumps must be carefully addressed in both the theoretical development and the stability analysis.

To overcome these difficulties, we establish a set of dual dynamic principles and verification theorems that connect the optimal control formulation with the desired stability properties. These results offer new insights into the mechanisms through which impulsive actions affect convergence rates and stability guarantees. Furthermore, the developed framework highlights the distinct roles that continuous control actions and impulsive interventions play in achieving finite-time and fixed-time stabilization.

The effectiveness and practical relevance of the proposed methodology are illustrated through representative examples. These examples show that impulses are not merely auxiliary features of the system but can be essential components in preserving and enhancing stability. In particular, they demonstrate how appropriately designed impulsive actions can significantly improve stabilization performance and, in some cases, are indispensable for ensuring the desired stability behavior within prescribed time bounds. Consequently, the proposed framework provides both a novel theoretical perspective and a practical tool for the analysis and design of impulsive dynamical systems.

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Well-posedness analysis of a unilateral contact problem

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We consider a mathematical model that describes the equilibrium of a nonlinear elastic membrane which can arrive in contact with a rigid obstacle, the so-called foundation. The membrane is fixed on its boundary, is acted upon by a vertical force, the contact is frictionless and is modelled with the well-known Signorini boundary condition. The novelty is that we model the material's behaviour with a Hencky-type elastic constitutive law and, therefore, the problem is governed by a p -Laplacian operator. Using such a constitutive assumption makes the model nonstandard from a mechanical point of view and challenging from a mathematical point of view. We use a Weierstrass-type minimization argument to prove the unique weak solvability of the model. Then, we turn to the well-posedness analysis of the problem, which represents the main novelty of this manuscript. Thus, we introduce three well-posedness concepts, compare them and state and prove strong and weak well-posedness results. We also discuss the problem of finding an optimal well-posedness concept. Finally, we apply these results to prove that the solution depends continuously with respect to the density of applied forces and the initial gap.

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On nonlinear mechanics of 1-D elastic structures

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Nonlinear mechanics of 1-D elastic structures is effectively investigated in a geometrically consistent 4-D spacetime framework [1]. The 4-D trajectory manifold becomes the geometric object of interest on which motion is experienced. The Rate Virtual Power Principle is exploited and the variationally effective stress rate leveraged [2]. Distinction between material and spatial fields is clarified and material tensors are compared in a natural way by push along the motion. Constitutive relations are formulated as instantaneous incremental responses to a finite set of material tensor state variables and to their time convective rates along the motion. A computationally efficient incremental solution procedure is implemented to effectively address problems of 1-D elastic structures undergoing large displacements [3]. Elastic and non-elastic effects are considered. Numerical results for emblematic structural schemes are presented. Contributed outcomes support implementation of the presented approach for resolution of geometrically nonlinear structural problems, circumventing long-known and debated issues affecting finite deformation approaches [4].

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